Exercise 42

Fanciful shapes can be created by using the implicit plotting capabilities of computer algebra systems.

(a) The curve with equation

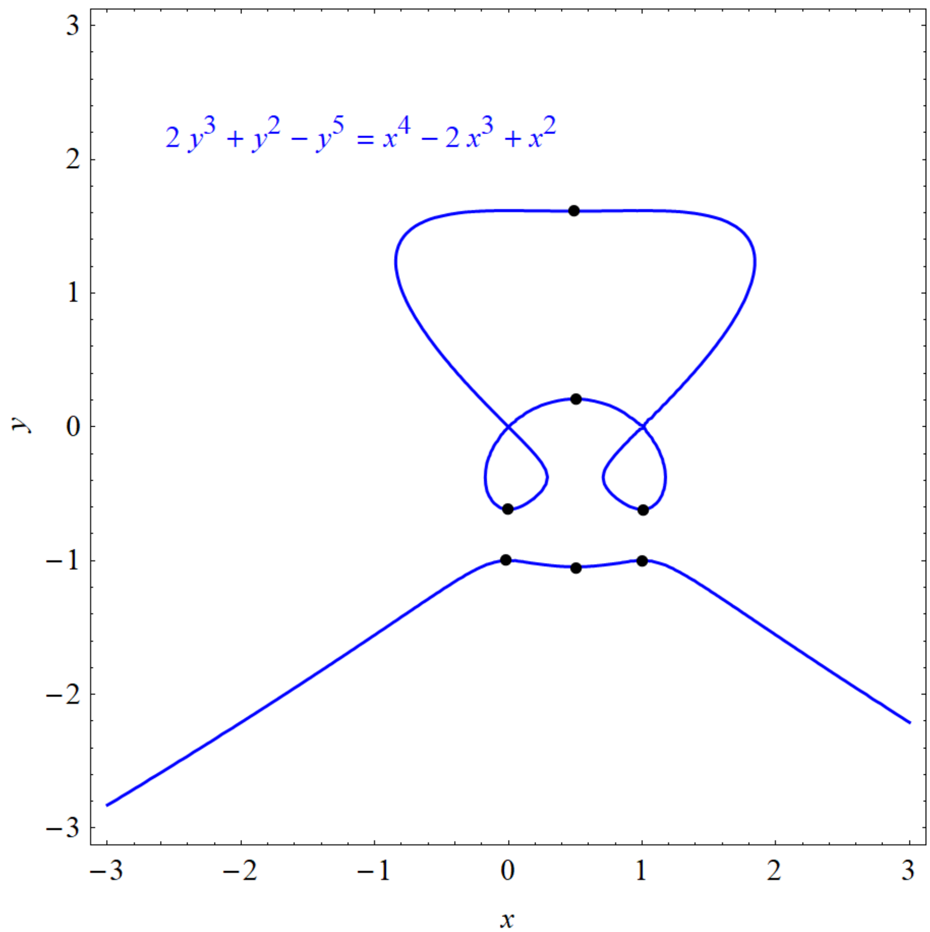
$$2y^3 + y^2 - y^5 = x^4 - 2x^3 + x^2$$

has been likened to a bouncing wagon. Use a computer algebra system to graph this curve and discover why.

(b) At how many points does this curve have horizontal tangent lines? Find the x-coordinates of these points.

Solution

Below is a graph of the curve.



There are seven points where the tangent line is horizontal.

Differentiate both sides with respect to x and then solve for y'.

$$\frac{d}{dx}(2y^3 + y^2 - y^5) = \frac{d}{dx}(x^4 - 2x^3 + x^2)$$

$$(6y^2)y' + (2y)y' - (5y^4)y' = 4x^3 - 6x^2 + 2x$$

$$y' = \frac{4x^3 - 6x^2 + 2x}{6y^2 + 2y - 5y^4}$$

To find the x-coordinates of the points where the tangent line is horizontal, set $y' = 0$ and solve for x.

$$y' = \frac{4x^3 - 6x^2 + 2x}{6y^2 + 2y - 5y^4} = 0 \quad \rightarrow \quad 4x^3 - 6x^2 + 2x = 0$$

$$4x\left(x - \frac{1}{2}\right)(x - 1) = 0$$

$$x = \left\{0, \frac{1}{2}, 1\right\}$$